Using Brewster angle for measuring microwave material parameters of bi-isotropic and chiral media

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ABSTRACT: This contribution focuses on the retrieval of chiral and nonreciprocal material parameters of bi-isotropic media. Using the generalized Fresnel reflection coefficients, that the authors have recently derived for general bi-isotropic media, a reflection method is suggested for determining the materials parameters of an unknown material sample. The sample needs to be thick enough for no transmission and multiple reflection effects to occur, and it should have one planar surface extending widely enough to cover the beam of the measuring antenna beam.

1. INTRODUCTION

Recent times have witnessed increasing interest towards complex materials in microwave and millimeter wave studies, due to their possible applications in radomes, antennas, reflectionless coatings, phase shifters, etc. (see, for example Jaggard. Liu, and Sun: Spherical chiroshield, *Electronics Letters*, Vol. 27, No. 1, p. 77-79, 1991; Lindell, Sihvola, Viitanen, Tretyakov: Geometrical optics in inhomogeneous chiral media with application to polarization correction in inhomogeneous lens antennas, *Journal of Electromagnetic Waves and Applications*, Vol. 4, No. 6, p. 533-548, 1990). Among these are chiral and nonreciprocal media. These can be seen as examples of general bi-isotropic media that have to be described through four scalar material parameters, and can be modeled, for example, with the following constitutive relations

$$\bar{D} = \epsilon\bar{E} + (\chi - j\kappa)\sqrt{\mu_0\epsilon_0}\bar{H} \quad (1)$$

$$\bar{B} = \mu\bar{H} + (\chi + j\kappa)\sqrt{\mu_0\epsilon_0}\bar{E} \quad (2)$$

which give the magnetoelectric relation between the electric $\bar{E}$ and magnetic fields $\bar{H}$, and the electric $\bar{D}$ and magnetic flux densities $\bar{B}$. In addition to permittivity ϵ and permeability μ, the dimensionless cross-coupling terms are the chirality (or handedness) parameter κ and the nonreciprocity parameter χ. For the case $\chi = 0$, the material is reciprocally chiral (in the sequel this type of medium is referred to as *Pasteur* medium), and in case $\kappa = 0$, it is nonreciprocal, but nonchiral, or *Tellegen* medium. j is the imaginary unit, indicating the time-harmonic dependence $\exp(j\omega t)$ and μ_0, ϵ_0 are the permeability and the permittivity of free space.

Recent progress in material manufacturing has made it clear that very soon chiral composites are available; i.e. mixtures of metal helices or handed polymer structures with a host material, which exhibit "optical" activity at microwave or millimeter wave frequencies (a better term would then be "electromagnetic activity"). Handed structures, as is known, produce rotation of the plane of the linearly polarized wave. The difference between this rotation effect and the earlierly known Faraday rotation is that the rotation due to chirality is isotropic and reciprocal, and due to the geometry of the medium itself; whereas Faraday rotation is due to an external magnetic field, therefore rendering the material unavoidably anisotropic. We have suggested the use of the term Pasteur medium to the chiral reciprocal material in honor of Louis Pasteur, who in 1840's connected the earlierly known optical activity to the microstructure of the solutions he studied (tartaric and racemic acids).

The other novel parameter in the microwave characterization of complex media, the nonreciprocity parameter χ was first suggested by Tellegen (*Philips Research Reports,* 3, 81-101, 1948) as he proposed a new network element to circuit theory, the gyrator. This parameter is seen to affect especially the reflection properties of the electromagnetic wave, in contrast to the propagation-affecting abilities of the chirality parameter κ. However, for a plane wave obliquely incident onto the surface of a bi-isotropic interface, both parameters (in addition to the classical ones, ϵ and μ) affect. This suggests also the possibility to measure these through reflection measurements.

This presentation will present our latest studies on the reflection problem of plane waves from an isotropic–bi-isotropic planar interface. We have managed to derive for the first time the generalized Fresnel reflection coefficients for this problem (Helsinki University of Technology, Electromagnetics Laboratory Report Series, Report 100, September 1991; for a communication note, see *Microwave and Optical Technology Letters,* Vol. 5, No. 2, p. 79-81, February 1992). The use of these results suggests a new way of measuring all four material parameters of a general bi-isotropic medium sample. Especially promising seems to be the use of the Brewster angle to this material characterization.

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2. PLANE-WAVE REFLECTION FROM A BI-ISOTROPIC INTERFACE

In the problem of the reflection of a plane wave from the planar interface between two isotropic media that only differ by their permittivity and permeability, the eigenpolarizations (i.e. polarization states of the incident field that do not change polarization in reflection) are vertically and horizontally linearly polarized waves. This is not the case anymore in reflection from a general bi-isotropic interface.

Therefore, if the incident field that is propagating towards the interface between an isotropic and bi-isotropic half space, is either horizontally or vertically polarized, it is not anymore enough to characterize the reflection by one scalar reflection coefficient. An incident horizontally polarized wave will give rise to both co- and cross-polarized reflections, and vice versa (note, however, that there appear also nonreciprocal phenomena due to the Tellegen parameter).

Consider the geometry of Figure 1, which defines the reflection problem. We define as the eigenpolarizations those polarizations that preserve their states in reflection; these are elliptical, in general. Let the plane wave impinge from the upper half space with parameter ϵ_0, μ_0 to the lower one (bi-isotropic) with parameters $\epsilon, \mu, \kappa, \chi$. The angle of incidence (the angle between the incident wave vector and the normal of the interface plane) is θ_0, which is also equal to the angle of reflection. There are two refracted rays, because the eigenwaves in a bi-isotropic medium propagate with two different phase velocities. According to the Snell's law, the refracted angles θ_+, θ_- obey the relation

$$\sin\theta_0 = n_+ \sin\theta_+ = n_- \sin\theta_- \tag{3}$$

with

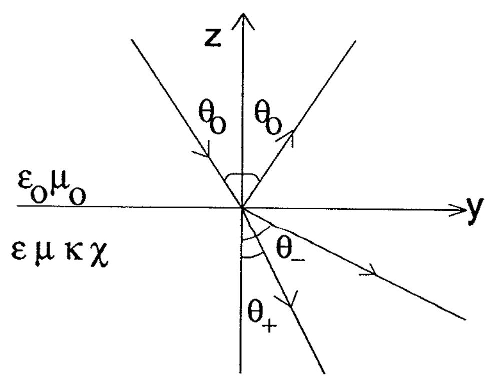

Figure 1 The geometry of the problem: a plane wave is incident on the planar interface between an isotropic and a bi-isotropic medium. There are two refracted rays, and one reflected ray.

$$n_\pm = n\cos\vartheta \pm \kappa, \quad n = \sqrt{\frac{\mu\epsilon}{\mu_0\epsilon_0}}, \quad \vartheta = \arcsin\left(\frac{\chi}{n}\right) \tag{4}$$

The eigenvalues for the reflection problem are the two generalized Fresnel reflection coefficients:

$$R_{1,2} = \frac{\eta P_{1,2} - \eta_0}{\eta P_{1,2} + \eta_0} \tag{5}$$

where the impedances of the two half spaces are $\eta = \sqrt{\mu/\epsilon}$ and $\eta_0 = \sqrt{\mu_0/\epsilon_0}$, and

$$P_{1,2} = \frac{1}{P_{2,1}} = Q\cos\vartheta \pm \sqrt{Q^2\cos^2\vartheta - 1} \tag{6}$$

$$Q = \frac{\xi_+\xi_- + 1}{\xi_+ + \xi_-}, \quad \xi_\pm = \frac{\cos\theta_\pm}{\cos\theta_0} \tag{7}$$

It is a straightforward exercise to evaluate from these the classical Fresnel coefficients, as the bi-isotropic half space degenerates into an isotropic one, $\kappa \to 0$, $\chi \to 0$ ($\vartheta \to 0$), whence $\cos\theta_+ \to \cos\theta$, $\cos\theta_- \to \cos\theta$:

$$R_1 \to \frac{\eta\cos\theta - \eta_0\cos\theta_0}{\eta\cos\theta + \eta_0\cos\theta_0} = R_{\parallel} \quad \text{(vertical polarization)} \tag{8}$$

$$R_2 \to \frac{\eta\cos\theta_0 - \eta_0\cos\theta}{\eta\cos\theta_0 + \eta_0\cos\theta} = R_{\perp} \quad \text{(horizontal polarization)} \tag{9}$$

Also, the reciprocal chiral case (Pasteur medium), that has been studied by Lakhtakia et al. (A parametric study of microwave reflection characteristics of a planar achiral-chiral interface, *IEEE Trans. Electromagnetic Compatibility*, Vol. 28, No. 2, p. 90-95, May 1986), and Bassiri et al. (Electromagnetic wave propagation through a dielectric-chiral interface and through a chiral slab, *Journal of the Optical Society of America* A, Vol. 5, No. 9, p. 1450-1459, Sept. 1986), can be seen to follow as a special case from our result.

3. BREWSTER ANGLES

The concept of Brewster angle can also be generalized to the isotropic–bi-isotropic reflection problem. Brewster angle is not plainly interpreted as the zero-reflection angle for the vertically polarized incident field, but rather it is the polarizing angle. (For discussion on Brewster angle regarding complex media, see for example, A. Lakhtakia: Would Brewster recognize today's Brewster angle? *OSA Optics News*, Vol. 15, No. 6, p. 14-18, 1989.) This Brewster angle is the angle θ_0 for which any incident field will be completely polarized in reflection. This is the case when either of the reflection coefficients vanish. This requirement leads to

$$\tan^2\theta_B = \pm\frac{4R[R\pm(C_+/C_-)][R\pm(C_-/C_+)]}{C_+C_-(R^2-1)^2}, \qquad (10)$$

with

$$C_+ = \sqrt{1-(1/n_+)^2}, \quad C_- = \sqrt{1-(1/n_-)^2}, \qquad (11)$$

$$R = \sqrt{\frac{(\eta_+-\eta_o)(\eta_--\eta_o)}{(\eta_++\eta_o)(\eta_-+\eta_o)}}. \qquad (12)$$

This expression can be tested for the isotropic interface, in which case, by setting $C_+ = C_- = C = \sqrt{(\mu_r\epsilon_r-1)/\mu_r\epsilon_r}$ and $R = (\eta-\eta_o)/(\eta+\eta_o)$, we have the two solutions

$$\tan^2\theta_B = \pm\frac{4R}{C^2(R\mp 1)^2} = \frac{\mu_r\epsilon_r}{\mu_r\epsilon_r-1}[(\mu_r/\epsilon_r)^{\pm 1}-1], \quad (13)$$

or

$$\tan\theta_{B1} = \sqrt{\frac{\mu_r(\mu_r-\epsilon_r)}{\mu_r\epsilon_r-1}}, \qquad \tan\theta_{B2} = \sqrt{\frac{\epsilon_r(\epsilon_r-\mu_r)}{\mu_r\epsilon_r-1}}, \quad (14)$$

which coincide with the well-known results.

Also, the reciprocal chiral special case, which does not seem to have been solved earlier in analytic form, emerges from this result.

Numerical results are shown In Figures 2 and 3. The Brewster angles are illustrated for some material parameter values of the bi-isotropic half space. From these it is seen that as chirality increases, the Brewster angle also increases, and attains the value 90° (grazing angle) as the chirality parameter satisfies

$$\kappa = n - 1. \qquad (15)$$

Interestingly enough, slightly before this value a second Brewster angle appears, and a qualitatively new phenomenon is predicted compared to the isotropic problem: the existence of two Brewster angles for a single material interface as can be seen from the figures. This second Brewster angle is a very sensitive function of the chirality parameter, spanning the whole range of angles ($0° < \theta_B < 90°$) within a small regime of chirality. This suggests an accurate means of measuring the chirality of a planar material through reflection ellipsometry methods. Of course, it should be admitted that the two Brewster angles only appear for a certain range of the combination of the four material parameters, and therefore this extremely accurate method (witnessed by the steepness of the second Brewster angle curve in Figures 2 and 3) cannot be exploited in all cases. If the parameters fall outside this range, then only information from one Brewster angle is available, and to get sufficient amount of measured data, also the reflection amplitudes for different polarizations are needed.

The effect of the Tellegen parameter χ on the Brewster angles can be studied from Figures 2 and 3. It is seen that as either the chirality or the Tellegen parameter are increased, the Brewster angle also increases. For larger χ values, the grazing Brewster is attained earlier, i.e. for smaller chirality values. It can also be seen from Figure 3 that for higher Tellegen parameter values the range of the second Brewster angle will be broader.

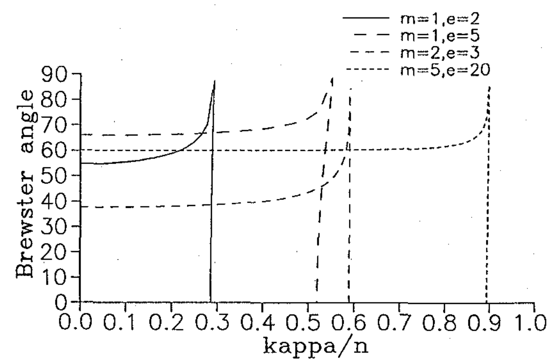

Figure 2 The dependence of the Brewster angle on the normalized chirality parameter κ/n of the lower half space in the problem of a nonchiral–chiral planar interface, where the lower half space is reciprocal. $n = \sqrt{\mu\epsilon/\mu_0\epsilon_0}$ is its refractive index (relative to the upper half space). Shown are four different permittivity and permeability cases of the lower half space: $(\mu/\mu_0, \epsilon/\epsilon_0) = (1,2); (1,5); (2,3); (5,20)$. Note the appearance of two Brewster angles in a short range close to the upper limit in the chirality parameter.

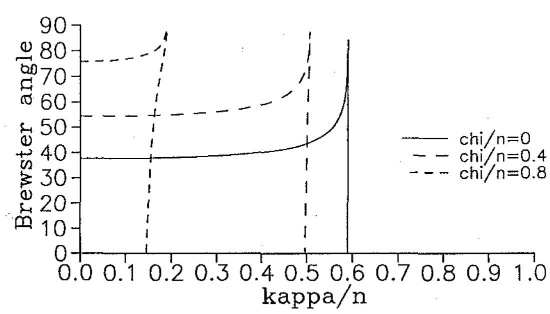

Figure 3 The dependence of the Brewster angle on the normalized chirality parameter κ/n of the lower half space in the problem of a nonchiral–bi-isotropic planar interface, where the lower half space is nonreciprocal. The permittivity of the lower half space is $3\epsilon_0$ and its permeability is $2\mu_0$. Shown are three Tellegen parameter values: $\chi/n = 0$ (reciprocal case), 0.4, and 0.8.

The previous Brewster angle guarantees that no power is reflected in the corresponding eigenpolarization. If, on the other hand, the reflection is described in terms of vertically and horizontally polarized field components (not eigenpolarizations in the bi-isotropic case), there appears a reflection matrix with vv, hh, vh, and hv components. Interesting may be the nonreciprocal phenomenon in the reflection where one of the crosspolarization reflection coefficients vanishes and the other still maintains a finite value. This naturally demands that the Tellegen parameter of the lower half space be nonzero.

The requirement for this kind of nonreciprocal "Brewster angle" is that $R_{hv} = 0$ (corresponding to the case that a vertically polarized incident field gives rise to no horizontally polarized reflected field, but not vice versa). From this requirement the material parameters can be calculated. The result is that the Tellegen parameter should have the value $\chi = n/\sqrt{2}$ (and, in the parallel case $R_{vh} = 0$ it should have the value $\chi = -n/\sqrt{2}$). The requirement for the Pasteur parameter κ, on the other hand, depends on the refractive index, but is the same for both cases $R_{hv} = 0$ and $R_{vh} = 0$. The dependence of this cross-Brewster angle on the chirality parameter is shown in Figure 4 for certain refractive index values.

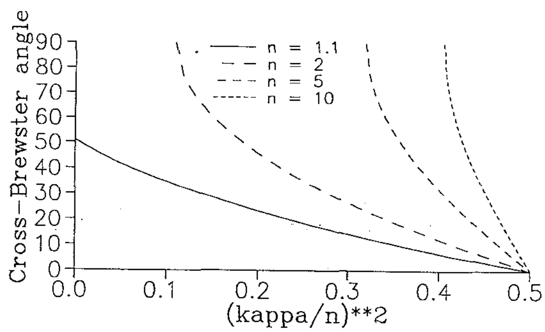

Figure 4 The dependence of the cross-Brewster angle (the angle for which $R_{hv} = 0$ or $R_{vh} = 0$) on the chirality for certain refractive index values of the lower half space. The axis parameter is $(\kappa/n)^2$. The other Brewster requirement is that for the Tellegen parameter: $\chi = n/\sqrt{2}$ for $R_{hv} = 0$ and $\chi = -n/\sqrt{2}$ for $R_{vh} = 0$.